QUESTION 1 (12 Marks) **Marks**

(a) Consider the parabola $(x-4)^2 = 8(y+3)$.

(i) State the co-ordinates of the vertex **1**

(ii) Find the focal length. **1**

(iii) Find the co-ordinates of the focus **1**

(iv) Find the equation of the directrix. **1**

(b) Re-write $y = x^2 + 4x + 3$ in the form $(x-h)^2 = 4a(y-k)^2$. **2**

(c) A parabola has its focus at the point (2, −1) and directrix $y = 3$.

Find

(i) the focal length. **1**

(ii) the co-ordinates of the vertex. **1**

(iii) the equation of the parabola. **2**

(d) Sketch the locus of the point *P(x,y),* where *P* is 3 units from the point *A*(3,1) and hence write down its equation, **2**

QUESTION 2 *(14 Marks)* Start this question on a new page.

(a) Consider the curve $y = \frac{x^3}{3} - 4x$

(i) Show that $\frac{dy}{dx} = x^2 - 4$ **1**

(ii) Find the points where the curve crosses the axes. **2**

(iii) Find the coordinates of any stationary points and determine their nature. **3**

(iv) Find any points of inflection. (Change in concavity must be shown.) **2**

(v) Draw a graph of this function (about ½ page.) **2**

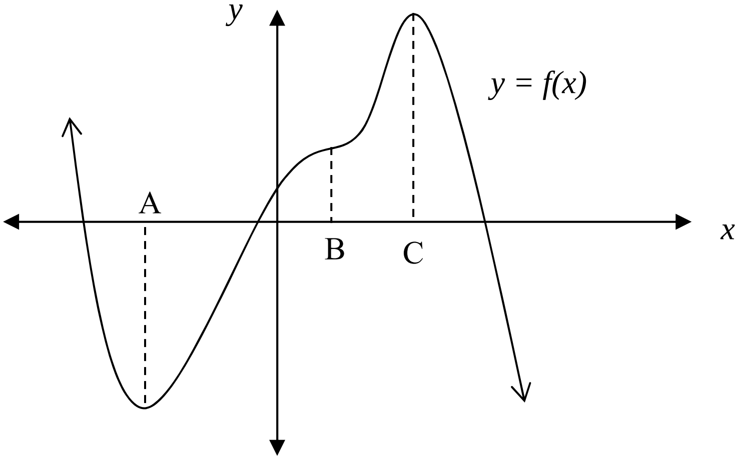
(vi) Find the equation of the tangent to this curve at the point $(3,-3)$. **2**

(b) Sketch the graph of $y = f(x)$ such that **2**
$f(3) = 5$, $f'(3) = 0$, $f'(x) > 0$ for $x < 3$ and $f'(x) < 0$ for $x > 3$.

Continued page 2

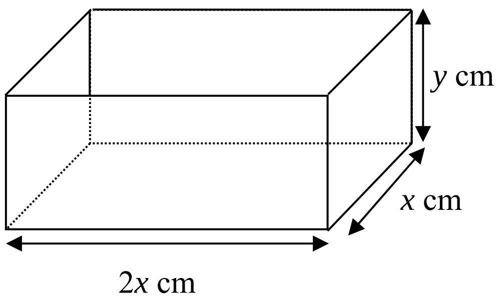
QUESTION 3 *(12 marks)* Start this question on a new page. **Marks**

(a) **3**



Draw or trace the graph $y = f(x)$, above, on your writing paper. On the same diagram, draw a graph of $y = f\,'(x)$, the derivative of the function, indicating the points A, B and C.

(b) An open rectangular box has four sides and a base, but no lid, as in the figure below.



The dimensions of the base of the box are x cm by $2x$ cm and the height is y cm.

(i) Write down the formula for the external surface area A cm^2 of the box in its simplest form. **2**

(ii) Write down the formula for the volume V cm^3 contained by the box in its simplest form. **2**

(iii) Given that the surface area, A, of the box is 150 cm^2, show that the formula for the volume in terms of x is $V = \dfrac{150x - 2x^3}{3}$. **2**

(iv) Find the value of x for which V is a maximum, and verify that the maximum value of V is $\dfrac{150}{3}$ cm^3. **3**

End of Paper.

11 Mathematics 2007 Yearly

Question 1

(a)(i) $(x-4)^2 = 8(y+3)$

So at vertex:

$x-4=0$

$x=4$

$y+3=0$

$y=-3$

$\therefore$ vertex at $(4,-3)$ ①

(ii) $4a = 8$

$a = 2$

$\therefore$ focal length is 2 ①

(iii) focus is a units above vertex (concave up)

$\therefore$ focus is $(4,(-3+2))$

$=(4,-1)$ ①

(iv) directrix is a units below vertex

$\therefore$ directrix is $y=-3-2$

i.e. $y=-5$ ①

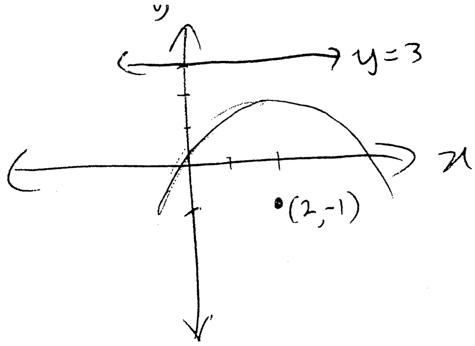
(b) $y=x^2+4x+3$

So $x^2+4x+4 = y+1$

$$\left.\begin{aligned}(x+2)^2 &= y+1 \\ &= 4\left(\tfrac{1}{4}\right)(y+1)\end{aligned}\right\}$$ ① for RHS

① for LHS

(c)



(i) Distance from focus to directrix is $2a$

So $2a = 3-(-1)$

$= 4$

$a = 2$

$\therefore$ focal length is 2. ①

(ii) The parabola is concave down.

The vertex halfway between the focus and directrix.

i.e. at $\left(2, \frac{-1+3}{2}\right)$ or focus plus a units up.

$= (2, 1)$ ①

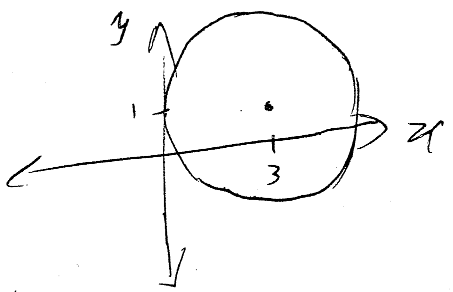
(iii) $(x-h)^2 = 4a(y-k)$

sub. in vertex and focal length.

$$\left.\begin{aligned} (x-2)^2 &= -8(y-1) \\ x^2 - 4x + 8y - 4 &= 0 \end{aligned}\right\} \text{either } ②$$

-1 mark if minus sign left out for concave down

(d)



① circle must touch y-axis

$$\left.\begin{aligned} (x-3)^2 + (y-1)^2 &= 9 \\ x^2 - 6x + y^2 - 2y + 1 &= 0 \end{aligned}\right\} \text{either } ①$$

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QUESTION 2

a) i) $y = \frac{x^3}{3} - 4x$

$\therefore y' = \frac{3x^2}{3} - 4$

$y' = x^2 - 4$ (1)

ii) $\frac{x^3}{3} - 4x = 0$

$x\left(\frac{x^2}{3} - 4\right) = 0$

$\therefore$ $x = 0$ or $\frac{x^2}{3} - 4 = 0$
(1)

$x^2 = 12$

$x = \pm 2\sqrt{3}$
(1)

$\therefore$ Crosses at $-2\sqrt{3}$, 0, $2\sqrt{3}$

iii) Stationary Points when $y' = 0$

$\therefore x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$ (1)

Also $y'' = 2x$

$\therefore y''(2) = 4$ $\quad$ $y''(-2) = -4$

> 0 $\quad$ < 0

$\therefore$ Min $\quad$ $\therefore$ Max (1)

$\therefore \left(2, -5\frac{1}{3}\right)$ is a minimum turning point

$\left(-2, 5\frac{1}{3}\right)$ is a maximum turning point (1)

iv) Points of inflexion when $y'' = 0$

$\therefore 2x = 0$

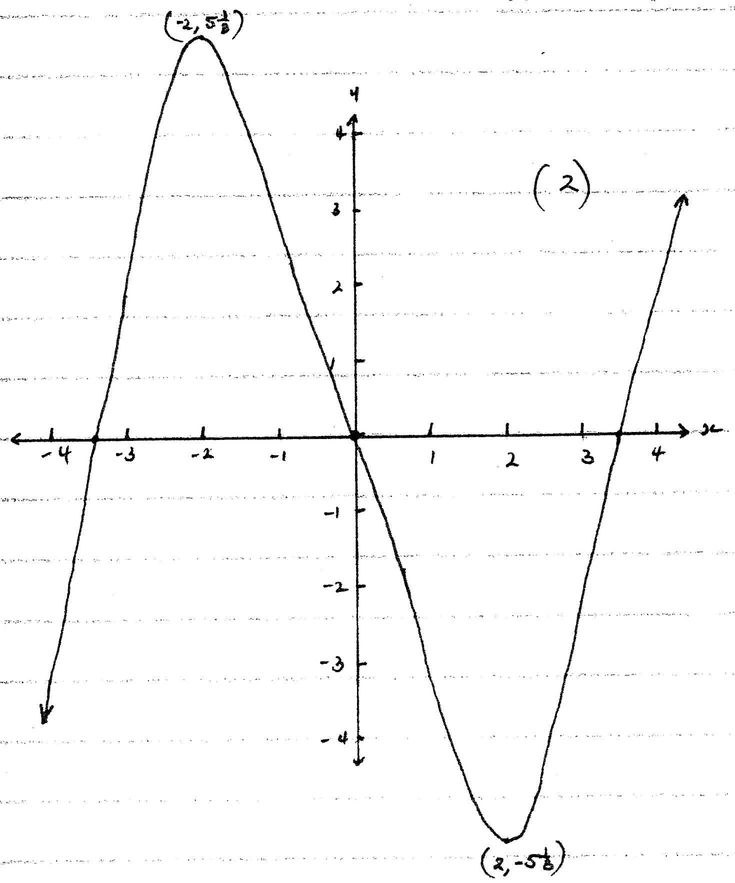
$x = 0$ (1)

x	-1	0	1
$f''(x)$	-2	0	2
	<0		>0

(1)

$\therefore$ Since concavity changes $(0,0)$ is a point of inflexion

v)



(2)

vi) Given $(3, -3)$

$y' = x^2 - 4$ when $x = 3$

$y' = 9 - 4$

$y' = 5$ (1)

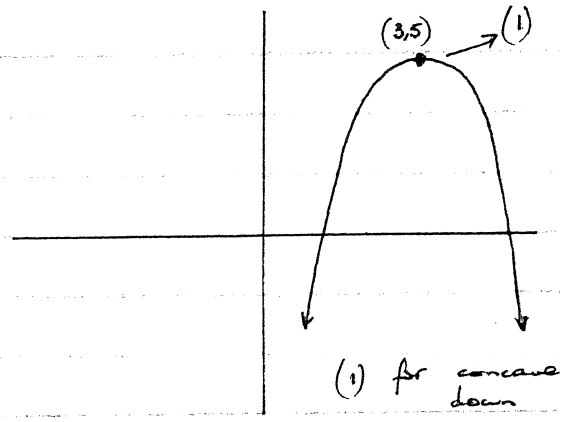
$\therefore m = 5$ $\quad$ $(3, -3)$

$y + 3 = 5(x - 3)$

$y + 3 = 5x - 15$

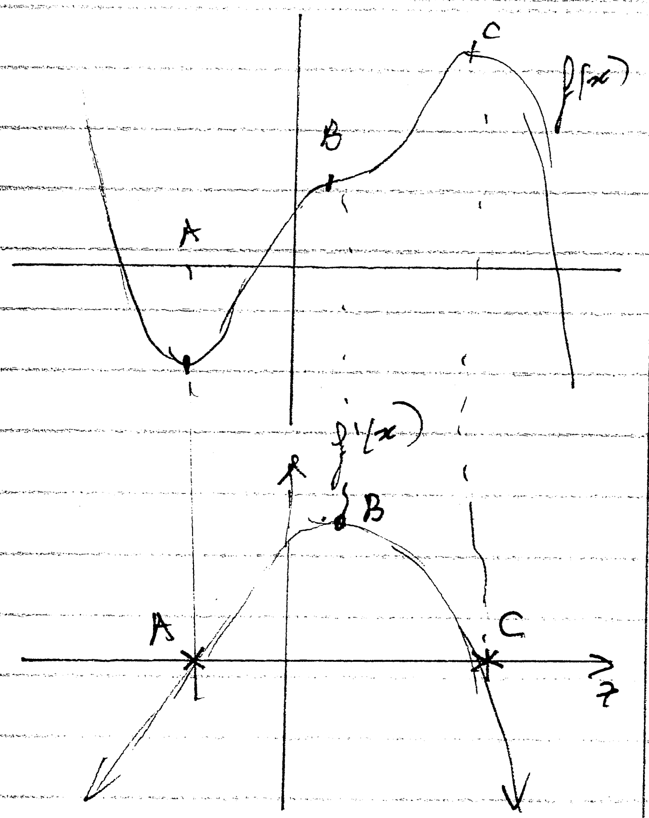
$y = 5x - 18$ (1)

b)



(1) for concave down

QUESTION THREE



(b) Surface Area =

$$2x(x.y) + 2(2x.y) + (2x.x)$$

sides Front/Back bottom.

$$A(x,y) = 2xy + 4xy + 2x^2$$

$$\underline{A(x,y) = 6xy + 2x^2}$$

(ii)

Volume $= L \times w \times h$

$$V(x,y) = (2x).(x).(y)$$

$$\underline{V(x,y) = 2x^2 y}$$

(iii)

Given $A = 150 = 6xy + 2x^2$

$$\therefore 6xy = 150 - 2x^2$$

$$y = \frac{150 - 2x^2}{6x}$$

$$= \frac{\cancel{2}(75 - x^2)}{3\cancel{2}x}$$

$$\underline{y = \frac{75 - x^2}{3x}}$$

Sub into V

$$V(x,y) = 2x^2 y$$

$$V(x) = 2x^2\left[\frac{75 - x^2}{3x}\right]$$

$$= \frac{2x^{\cancel{2}}}{3\cancel{x}}\left[75 - x^2\right]$$

$$\underline{V(x) = \frac{150x - 2x^3}{3}}$$

QED.

For MAX/MIN Find $V'(x) = 0$

$$V(x) = \frac{150x}{3} - \frac{2x^3}{3}$$

$$V'(x) = \frac{150}{3} - 2.\frac{\cancel{3}x^2}{\cancel{3}}$$

$$V'(x) = \frac{150}{3} - 2x^2 = 0$$

For MAX/MIN

$\therefore$ at $2x^2 = \frac{150}{3}$

$x^2 = \frac{25}{3}$ $\quad x^2 = 25$

~~$x = \pm\frac{5\sqrt{3}}{3}$~~ $\quad x = \pm 5$

Since $x > 0$

$\therefore x = \cancel{\frac{5\sqrt{3}}{3}}$ 5

~~$V\left(\frac{5\sqrt{3}}{3}\right) = \frac{150}{3}.\left(\frac{5\sqrt{3}}{3}\right) - \frac{2}{3}\left(\frac{5\sqrt{3}}{3}\right)^3$~~

~~$= \frac{150}{3}$~~

$$V(5) = \frac{150(5) - 2(5)^3}{3}$$

$$= \frac{500}{3}.$$